Binary arithmetic sample problems

8-bit additions:

10101011 11000011 00111011

+ 00111111 + 01010101 + 10110010

11101010 1 00011000 \* 11101101 \* - overflow

8-bit subtractions:

10001000 00110011 11100000

- 01101011 - 00011100 - 01111111

00011101 00010111 01100001

Alternatively, convert second number to two’s complement negative

10001000 00110011 11100000

+ 10010101 + 11100100 + 10000001

00011101 00010111 01100001

Unsigned Multiplication

0110 \* 0011

|  |  |  |  |
| --- | --- | --- | --- |
| C | A | Q | M=0011 |
| 0 | 0000 | 0110 | Q0 = 0, Shift C|A|Q |
| 0 | 0000 |  | Q0=1, add M |
| 0 | 0011 | 0|011 | Shift C|A|Q |
| 0 | 0001 |  | Q0=1, add M |
| 0 | 0100 | 10|01 | Shift C|A|Q |
| 0 | 0010 | 010|0 | Q0=0, Shift C|A|Q |
| 0 | 0001 | 0010 | Done |

0110 \* 0110 = 00010010 (6 \* 3 = 18)

010011 \* 110010

|  |  |  |  |
| --- | --- | --- | --- |
| C | A | Q | M = 110010 |
| 0 | 000000 | 010011 | Q0 = 1, add M |
| 0 | 110010 |  | Shift C|A|Q |
| 0 | 011001 | 0|01001 | Q0 = 1, add M |
| 1 | 001011 |  | Shift C|A|Q |
| 0 | 100101 | 10|0100 | Q0 = 0, Shift C|A|Q |
| 0 | 010010 | 110|010 | Q0 = 0, Shift C|A|Q |
| 0 | 001001 | 0110|01 | Q0 = 1, add M |
| 0 | 111011 |  | Shift C|A|Q |
| 0 | 011101 | 10110|0 | Shift C|A|Q |
| 0 | 001110 | 110110 | Done |

010011 \* 110010 = 001110110110 (19 \* 50 = 950)

Booth’s Algorithm Multiplication

1110 \* 0110

|  |  |  |  |
| --- | --- | --- | --- |
| A | Q | Q-1 | M=0110, -M=1010 |
| 0000 | 1110 | 0 | Q0-1 = 00, Shift A|Q|Q-1 |
| 0000 | 0111 | 0 | Q0-1 = 10, subtract M (add –M) |
| 1010 | 0111 |  | Shift A|Q|Q-1 |
| 1101 | 0011 | 1 | Q0-1 = 11, Shift A|Q|Q-1 |
| 1110 | 1001 | 1 | Q0-1 = 11, Shift A|Q|Q-1 |
| 1111 | 0100 | 1 | Done |

1110 \* 0110 = 11110100 (-2 \* 6 = -12)

100111 \* 001101

|  |  |  |  |
| --- | --- | --- | --- |
| A | Q | Q-1 | M = 001101, -M = 110011 |
| 000000 | 100111 | 0 | Q0-1 = 10, subtract M (add –M) |
| 110011 |  |  | Shift A|Q|Q-1 |
| 111001 | 1|10011 | 1 | Q0-1 = 11, Shift A|Q|Q-1 |
| 111100 | 11|1001 | 1 | Q0-1 = 11, Shift A|Q|Q-1 |
| 111110 | 011|100 | 1 | Q0-1 = 01, add M |
| 001011 |  |  | Shift A|Q|Q-1 |
| 000101 | 1011|10 | 0 | Q0-1 = 00, Shift A|Q|Q-1 |
| 000010 | 11011|1 | 0 | Q0-1 = 10, subtract M |
| 110101 |  |  | Shift A|Q|Q-1 |
| 111010 | 111011 |  | Done |

100111 \* 001101 = 111010111011 (-25 \* 13 = -325)

Unsigned Division

1110 / 0100

|  |  |  |
| --- | --- | --- |
| A | Q | M = 0100 |
| 0000 | 1110 | Shift |
| 0001 | 110\_ | A < M, Q0 = 0, Shift |
| 0011 | 100\_ | A < M, Q0 = 0, Shift |
| 0111 | 000\_ | A >= M, Q0 = 1, subtract M from A |
| 0011 | 0001 | Shift |
| 0110 | 001\_ | A >= M, Q0 = 1, subtract M from A |
| 0010 | 0011 | Done |

1110 / 0100 = 0011 with 0010 / 0100 remainder (14 / 4 = 3 and 2 / 4)

011110 / 000100

|  |  |  |
| --- | --- | --- |
| A | Q | M = 000100 |
| 000000 | 011110 | Shift |
| 000000 | 11110|\_ | A < M, set Q0 = 0, Shift |
| 000001 | 1110|0\_ | A < M, set Q0 = 0, Shift |
| 000011 | 110|00\_ | A < M, set Q0 = 0, Shift |
| 000111 | 10|000\_ | A > M, set Q0 = 1, subtract M from A |
| 000011 | 10|0001 | Shift |
| 000111 | 0|0001\_ | A > M, set Q0 = 1, subtract M from A |
| 000011 | 0|00011 | Shift |
| 000110 | |00011\_ | A > M, set Q0 = 1, subtract M from A |
| 000010 | 000111 | Done |

011110 / 000100 = 000111 and 000010 / 000100 (30 / 4 = 7 and 2 / 4)